HYPERSONIC SPHERICAL EXPANSION OF A GAS WITH STATIONARY SHOCK WAVE

A. A. Bochkarev, A. K. Rebrov,
and S. F. Chekmarev

Results are presented of an experimental study of the structure of the flow and shock wave for hypersonic spherical expansion of a gas in a region with finite pressure with transition from hypersonic to subsonic flow in a shock wave. Such flow is realized if sonic or supersonic flow conditions with a pressure exceeding considerably the surrounding pressure are created on a spherical surface. In the experiments the gasdynamic source was a hollow sphere with perforated shell.

The possibility is shown of creating a stationary spherical shock wave for efflux from the spherical source into the reduced pressure region with large pressure differential.

The influence of the pressure ratio and level on the structure of the flow with a spherical shock wave in nitrogen is studied.

The essential dependence of the flow parameters ahead of the shock wave on the perturbations which penetrate from the surrounding space through the shock wave is shown.

The study made of the binary mixture flow structure showed the presence of significant barodiffusional separation of the components.

Notation: n' = particle volume density, p = pressure, r = radial distance from center of source, M = Mach number, k = adiabatic exponent, f = mole fraction of heavy component in binary gas mixture, a = speed of sound, ρ = density, η = dynamic viscosity, $n = n'/n_0'$ = relative density.

Subscripts denote conditions: * = at critical section, 0 = stagnation, ∞ = in surrounding space, s = on shock wave, k = in vacuum chamber.

As is well known, the problem of stationary ideal gas flow from a spherical source has two continuous solutions: supersonic, with monotonic decrease of the pressure to 0 as $r \to \infty$, and subsonic, with monotonic increase of the pressure from p_* to $[\frac{1}{2}(k+1)]^{k/(k-1)}\, p_*$ at infinity. They are satisfied by the relation

$$\left(\frac{r}{r_*}\right)^2 = \frac{1}{M}\left(\frac{k-1}{k+1}M^2 + \frac{2}{k+1}\right)^{\frac{1}{2}(k+1)/(k-1)} \tag{1}$$

Consequently, the solution within the framework of ideal gas theory for efflux into space with finite pressure

$$p_\infty < p_* = [(k+1)/2]^{k(1-k)}\, p'_0$$

must be discontinuous with transition through the shock wave from the supersonic branch of the source to the subsonic branch of a new source whose location is defined by the quantity p'_0/p_∞ and the radius r_* of the sonic flow sphere.

The Mach number M_1 ahead of the shock wave [and therefore the position of the shock wave according to (1)] is defined by the expression

$$\frac{p_\infty}{p_0'} = \left(\frac{k+1}{2}\right)^{\frac{k+1}{k-1}} \left(\frac{2}{k-1}\right)^{\frac{1}{k-1}}, \qquad M_1^{\frac{2k}{k-1}} \left(1 + \frac{k-1}{2} M_1^2\right)^{\frac{k}{1-k}} \left(\frac{2k}{k-1} M_1^2 - 1\right)^{\frac{1}{1-k}} \tag{2}$$

Novosibirsk. Translated from Zhurnal Prikladnoi Mekhaniki i Tekhnicheskoi Fiziki, Vol. 10, No. 5, pp. 62-67, September-October, 1969. Original article submitted May 16, 1969.

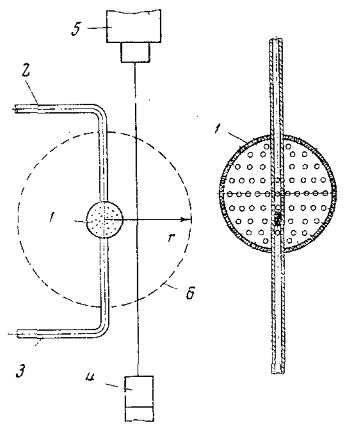

Fig. 1

The flow described can be realized in practice during gas expansion through a perforated or porous surface into a region with reduced pressure. It is possible that supersonic flow conditions will also be created during intense vaporization from the spherical surface into a vacuum.

In the studies described we used a perforated spherical shell – a 36-mm-diam. tennis ball in which 133 uniformly distributed 0.85-mm-diam. holes were drilled.

The experiments were conducted in a low-density wind tunnel having an output of 50,000 liters/sec at a pressure of $1 \cdot 10^{-2}$ Torr.

The gasdynamic source (tennis ball) was located in the 1600-mm-diam. working chamber. The purpose of the preliminary experiments was to verify the possibility of obtaining a spherical shock wave for conditions in which the pressure in the vacuum chamber varies from a few microns to a few tens of microns and the pressure inside the sphere is a few tens of Torr. Therefore in the first experiments only a glow discharge technique was used for flow visualization. The cathode was the copper tube used to supply the gas to the sphere and the anode was the shell of the chamber.

In the discharge with voltage 2.5 kV and current 5 mA, the zone adjacent to the sphere surface and the spherical layer far from the surface, corresponding to the high density in the shock wave, had increased brightness.

The flow near the surface of the source can be represented as efflux from an orifice with high pressure differential into the coflowing supersonic stream of the neighboring jets. The collision of the supersonic streams and their mixing may cause nonisentropic expansion, which leads to total pressure losses and reduction of the Mach number. For spherical shock wave studies it is necessary that the mixing zone be localized near the surface of the source.

A qualitative study of the spherical expansion preceded the quantitative study of the flow structure and the shock wave structure, based on the use of electron beam diagnostics for measuring the density.

A schematic of the test section is shown in Fig. 1. The perforated sphere 1 was mounted on a traversing mechanism on two tubes 2 and 3, which were used to supply the working gas and measure the pressure in the cavity of the sphere (4 is the electron gun, 5 is the electron collector, 6 represents the shock wave).

The electron beam generated by the gun 4 penetrated the entire flow region being tested. The mechanized displacement of the traversing mechanism made it possible for the electron beam to scan the half-space on one side of the source. The radiation excited by the electron beam was focused on the slit of an ISP-51 monochromator, equipped with a photographic recorder (FÉU-27 photomultiplier and ÉPPV-60 recorder). A segment of height 1.6 mm was segregated from the luminous column in the gas for voltages of 10 kV and current of 1-3 mA the beam diameter did not exceed 1.5 mm. This situation provided a local measurement.

Experiments were made using nitrogen, helium, argon, and mixtures of helium and argon. For the density measurements in helium we used the spectrum 5016 ± 10 Å, in argon 4200 ± 50 Å, and in nitrogen 4278 ± 16 Å.

The preliminary calibration for the density measurement was made under static conditions at a temperature of 300° K in the 7-45 μ Hg range; for all the gases the signal was linear with pressure.

The objective of the measurements was to study the position of the shock wave, the intensity of the density change across the shock, and the influence of various parameters on the density change in the supersonic region.

The influence of the pressure differential on the flow structure can be seen in Fig. 2, which shows the radial density profiles for the expansion of nitrogen. $n = n'/n'_0$ is the molecular density at the measurement point, referred to the molecular density in the source; the pressure p_0 inside the sphere remains

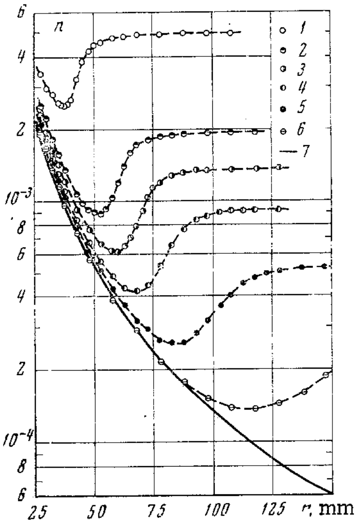

Fig. 2

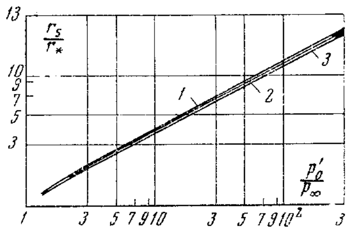

Fig. 3

constant at 17 Torr while the ambient pressure p_k varies in the range from 5.9 to 74 μ Hg; curves 1, 2, 3, 4, 5, 6 correspond to p_k = 0.0735, 0.0368, 0.0264, 0.0199, 0.0096, 0.0059 Torr, curve 7 is for the isentropic calculation. Here p_0/p_k varied from 2900 to 231. If we take the inflection point on the density curve as the shock wave position, this variation of p_0/p_k corresponds to reduction of the shock wave diameter from 285 to 85 mm, and the shock wave front becomes steeper.

The ratio of the densities, determined from their values behind the shock wave and at the density minimum, first increases somewhat with reduction of p_0/p_k and then decreases. The value of this ratio is close to two in all the regimes. The geometric Mach number is more than six in the observation zone ahead of the shock wave and in this case the density ratio (in the infinitely thin shock wave case) must be close to its limiting value $(k + 1)/(k - 1) = 6$. The significant difference between this value and the density ratio found from the experiment is explained by the interaction of two factors: the sphericity of the expansion in the shock wave itself and the deep upstream penetration of the disturbances. The latter is indicated by the marked increase of the density ahead of the shock wave with increase of the ambient pressure. Moreover, this disturbance propagates upstream practically to the source.

It is well known that a molecular velocity distribution which is non-Maxwellian is possible in hypersonic rarefied flow, which shows up in nonisentropicity of the molecular mean free paths [1]. The molecular mean free path in such nonequilibrium flow is longer in the longitudinal direction than in the transverse direction, which facilitates upstream penetration of the disturbances from the shock wave.

Since under our experimental conditions the shock wave zone and the mixing zone occupy radial dimensions comparable with the dimensions of the supersonic stream, the parameter differential across the shock wave differs significantly from that which follows from calculation using the Hugoniot adiabat and the formulas (1) and (2).

It is interesting to compare the shock wave location from the calculated data and the experimental results. In the limiting case when $M \to \infty$, it follows from the relations (1) and (2) for the shock wave radius r_s that

$$r_s / r_* \sim \sqrt{p_0' / p_\infty}$$

Figure 3 shows the dependence of r_s/r_* on p'_0/p_∞. For $M > 3$ approximately

$$r_s / r_* \approx a \sqrt{p_0' / p_\infty} \tag{3}$$

Curves 1, 2, 3 correspond to the values k = 1.25, 1.4, 1.67, for which a = 1.19, 1.27, 1.34, respectively. It follows from the experimental data (Fig. 2) that the shock wave radius (defined by the inflection point on the density curve) is

$$r_s = 2.84 \sqrt{p_0 / p_k}, \text{ mm} \tag{4}$$

which corresponds to the magnitude of the sonic-flow sphere radius r_* = 2.25 mm.

Figure 2 shows the density relation for isentropic expansion from this sphere.

The results of the study of the influence of rarefaction on the flow structure are presented in Fig. 4, where curves 4, 3, 2, 1 correspond to the parameters: p_0 = 34.5, 17.5, 8.45, 4.26 Torr, p_k = 0.0528, 0.0264, 0.0132, 0.0066 Torr.

The flow structure as a function of the pressure ratio p_0/p_k = 645. Analysis of the relations obtained makes it possible to state that in the pressure range $p_k = 5.3 \cdot 10^{-2}$–$6.6 \cdot 10^{-3}$ Torr there is marked displacement of the shock wave. Moreover, with increase of the pressure level, there is an increase of the relative density in the supersonic part of the flow, which can be explained by reduction of the viscous dissipation in the orifices of the source in the mixing zone, which is equiavlent to increase of the radius of the fictitious sonic-flow sphere.

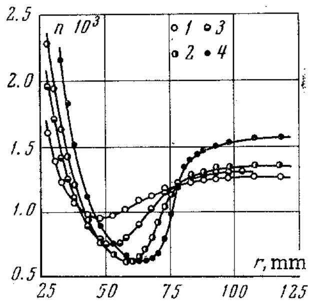

Fig. 4

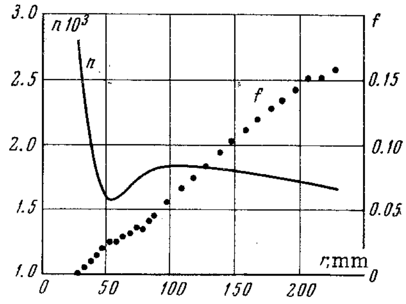

Fig. 5

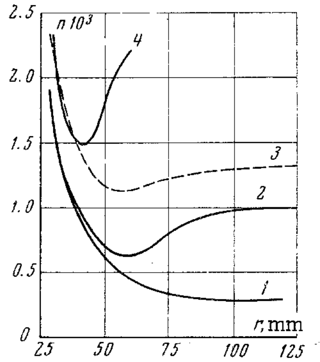

Fig. 6

The density ratio across the shock wave in the pressure range studied varied from 1.34 to 2.58.

The crossing of the resulting curves (in Fig. 4) at a single point in the shock wave zone may be simply a random occurrence.

Figure 5 shows the results of direct verification of the penetration of molecules from the surrounding space into the zone ahead of the shock wave. The experiment was conducted as follows: simultaneously with the efflux of helium from the perforated sphere, nitrogen was fed into the vacuum chamber. The pressure in the sphere cavity was 8.5 Torr, the pressure in the surrounding space was 0.015 Torr. Characteristic of the radial variation of the density in the heluim flow is a reduction of the density behind the shock wave.

The curve of radial variation of the nitrogen concentration indicates that the nitrogen molecules for the given experimental conditions penetrate practically to the surface of the gasdynamic source.

The radial variation of the nitrogen concentration is nearly linear except for the kinks at the shock wave.

The result obtained is of importance for evaluating the influence of the conditions in the surrounding space on the structure of hypersonic rarefied gas flows in a medium at rest.

The study of the flow structure of a binary mixture of monatomic gases was made using argon and helium. The relations for the density distribution of the individual gases were first obtained. In Fig. 6, curves 1, 2, 4 show the results of measurement of the density n for the case of argon expansion with backpressures p_k = 0.00324, 0.0107, 0.0243; here p_0 = 9.78 Torr. The pattern is qualitatively the same as for nitrogen (Fig. 2). The density ratio across the shock wave in the range studied does not exceed 1.6.

This same figure shows the results of measurements in helium flow (curve 3) for p_k = 0.0118, p_0 = 8.84 Torr. Comparison of the density ratio across the shock wave of helium and argon for similar parameters shows that the intensity of the shock wave in helium is considerably less, which is explained by the higher rarefaction of the helium flow for the same pressure.

It follows from the experimental data of Fig. 6 that for the selected source in the parameter range investigated the shock wave radius for the monatomic gases obeys the relation

$$r_s = 2.46 \sqrt{p_0 / p_k} \tag{5}$$

which corresponds to the value of the fictitious sonic-flow sphere radius $r_* = 2.05$ mm.

Figure 7 shows results of measurements of the partial density of the components and the mole fraction $f = \varphi(r)$ of the heavy component in the argon-helium mixture flow 45.5% Ar + 54.5% He with initial concentration f_0 = 0.455. Curves 1, 2, 3, 4 correspond to p_0 = 30.1, 31.3, 31.3, 15.0 Torr. p_k = 0.0132, 0.0199, 0.0488, 0.0221 Torr. In Fig. 7a, n is the ratio of the component atom density to the component partial density in the sphere (continuous curve is for argon, dashed is for helium). In Fig. 7b, f denotes the argon concentration

$$f = \frac{n'(\mathrm{Ar})}{n'(\mathrm{Ar}) + n'(\mathrm{He})}$$

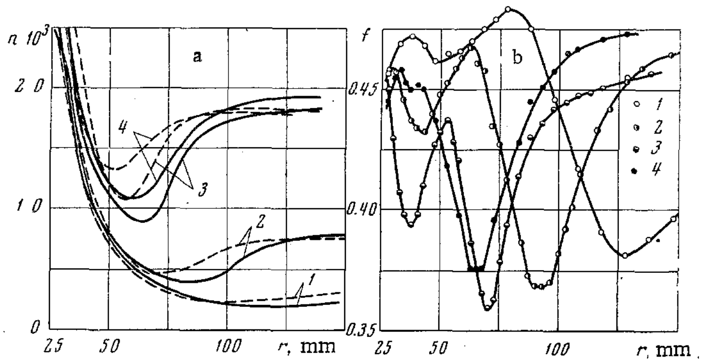

Fig. 7

Here n'(Ar) and n'(He) are the partial densities of the argon and helium atoms; the initial concentration level is 0.455.

A characteristic feature of all the regimes is the presence of barodiffusional separation of the gases with different molecular weights [2], which shows up in the fact that: a) the shock wave zone for the gas mixture is larger than for the individual gases; b) a high helium concentration is observed in the leading edge of the shock wave; c) behind the wave the concentration equalizes to the initial value.

Curves 1, 2, 3, recorded for constant p_0 and different p_k, show the effect of the ratio p_0/p_k. The shock wave radius, calculated using (5), corresponds approximately to the inflection points in the helium partial density profiles; the inflection points in the argon partial densities are shifted downstream somewhat. The increase of the argon partial density 1.8 in the shock wave (curve 4) is somewhat greater than for pure argon (Fig. 5) for similar values of the dimensional complex $a_0\rho_0/\eta_0$, where a_0, ρ_0, η_0 are the speed of sound, density, and dynamic viscosity in the stagnation chamber.

With reduction of p_k the shock wave zone which is rich in helium expands, while the maximal helium enrichment is reduced because of the reduction of the gradients. Reduction of the pressure level has the same effect (curves 3, 4).

The flow region for r < 50 mm corresponds to the zone of jet mixing and formation of the spherical flow. The different behavior of the concentration in this region under different conditions indicates the significant effect of the outer pressure and the location of the spherical shock wave on the mixing process. It is characteristic that with approach of the shock wave there is significant depletion of the heavy component (curves 2, 3).

In the case of supersonic acceleration of the gas, barodiffusional enrichment of the heavy component is observed downstream from the mixing zone.

Thus, under the conditions investigated the flow of the gases ahead of the shock wave is not in equilibrium. Therefore the quantitative data on the separation of gases at the shock wave are not simply a result of the barodiffusional processes in the shock wave itself. In order to study the shock wave structure in gas mixtures it is necessary to create conditions of lower flow rarefaction either by increasing the absolute density or by increasing the linear dimensions of the flow while maintaining the density level.

LITERATURE CITED

1. B. B. Hamel and D. R. Willis, "Kinetic theory of source flow expansion with application to the free jet," Phys. Fluids, Vol. 9, No. 5 (1966).
2. D. E. Rothe, Electron beam studies of the diffusive separation of helium-argon mixtures," Phys. Fluids, Vol. 9, No. 9 (1966).